The seeds of cosmic structure, the black hole information puzzle and the entropic arrow of time

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1 Introduction

The study of the interface between quantum theory and gravitation often takes the form of the search for a theory of quantum gravity; and although great strides have been made along various research lines adopted for that search, it is fair to say that, at this time, there is no fully satisfactory version of such a theory. Generally, the search for quantum gravity is viewed as independent of the conceptual problems afflicting quantum theory, namely, the lack of ontological clarity and the so-called measurement problem. There are, though, some exceptions to that general attitude. Several works exploring the interface between quantum theory and gravitation, in which conceptual quantum aspects are central to the discussion, have appeared in the literature over the years. Moreover, a few meetings devoted to such questions have taken place, specially over the last few years. In fact, the event that led to this book is, in a sense, one such example—even though, sometimes, questions regarding the nature of time are viewed as separated from the general relativistic context and, thus, from its intricate connection with gravitation. This is understandable because, from the philosophical point of view, the nature of time involves a large number of issues beyond those that arise in physics (issues which we do not discuss here).

In this work, we describe a research project that combines a particular strategy for the exploration of the gravity-quantum interface, with a specific point of view regarding foundational aspects of quantum theory. Moreover, we explain how the effort to adapt one to the other has led to a peculiar outlook regarding the entropic arrow of time. Our project takes general relativity (GR) as the (tentative) preferred description of gravitation and spacetime structure, while adopting a agnostic attitude towards the various existing approaches towards the construction of a true quantum theory of gravity. We, of course, acknowledge that, in all likelihood, a full theory of quantum gravity will be found and required but, at the same time, recognizing that we might be rather far from that point. Moreover, given that GR works rather well in the realms it has been tested, we work under the assumption that any reasonable theory of quantum gravity must have GR as a suitable approximation, under the appropriate circumstances. Regarding our treatment of matter, we take it as evident that it requires a quantum description and that the classical realm must be viewed only as an approximation—which we take, for the sake of our discussion, to be under relatively good control. In that sense, we take quantum field theory and, in particular, its curved space-time version, to provide our current best characterization of matter in general, and the standard model of particle physics as its concrete realization in our world.

These considerations drive us to adopt semiclassical gravity as the basic framework for our analysis. We cannot, then, ignore the objections against such a framework, particularly those raised by the experiment and analysis in Page and Geilker (1981). However, it has been pointed out that the issue is closely connected with the conceptual difficulties in quantum theory Carlip (2008); Huggett and Callender (2001); Mattingly (2005, 2006), which takes us to confront such issues head on. For this, we adopt the approach based on spontaneous collapse theories. It is well-know, however, that such frameworks display a lack of conservation of energy when a collapse is involved, which, as observed in Page and Geilker (1981), seems inconsistent with semiclassical gravity. The issue is substantially mitigated by taking semiclassical gravity as approximate, as we do, and by observing that as discussed in Maudlin et al. (2020), the problem of non-conservation seems to appear, in one guise or another, in all open avenues to deal with the measurement problem. As a result, it seems that our choice is not forcing us into a problem that could otherwise disappear.

Throughout, we try to be as conservative as possible, giving priority to well established theories when possible, aiming to find, often in connection with concrete examples, aspects of the various theoretical frameworks that give raise to tensions to be explored. We believe that, given the sheer complexity of the issues that arise in discussing a theory of spacetime itself, and the intrinsic difficulties of trying to discuss physics without a spacetime framework—together with the tendency to revert to our intuitions when facing the peculiar nature of the conceptual conundrums characteristic of quantum physics—makes essential the adoption of relatively clear postures as a means to prevent confusion and distraction from interfering with the sought out progress.

Our manuscript is organized as follows. In section 2, we describe in more detail our general approach, including a formalism that has been under development for some years devoted to the application of collapse models to semiclassical settings. Then, we present two specific scenarios to which we have applied such a formalism: the inflationary account for the emergence of the seeds of cosmic structure, in section 3, and the black hole information puzzle, in section 4. These explorations have provided us with interesting lessons which, in section 5, we employ to deal with the problem that concerns us most in this manuscript, the nature and origin of the entropic arrow of time. We show, in particular, how this approach offers an attractive way to implement some of the conjectures on the subject introduced by Roger Penrose in Penrose (1979). We close, in section 5, with a discussion.

2 The program

It seems quite difficult to propose a physical theory that does not contain some basic notions intrinsically tied to spacetime. Proponents of theoretical frameworks, where spacetime is supposed to emerge, must face this difficulty. This is so even when relying on classical notions, such as points, areas, volumes, curvature, etc.; but if the theory is to be formulated in a purely quantum mechanical language from he start, the emergence analysis become even more complex, as one can expect a rigorous treatment to require adopting a clear position regarding the conceptual difficulties of quantum theory.

In our opinion, a failure to acknowledge these issues often leads to confusion, as avoiding a (at least a temporary) commitment to a definite position regarding the difficult interpretative issues of quantum theory, removes self-consistency constrains or allows for specific words to change their meaning in the middle of the discussion. One rather common example is provided by the word "fluctuation", which is used to reflect various notions that are, in principle, quite distinct. For instance, on one hand, there are *quantum uncertainties*, which, according to the standard interpretation, represent levels of indeterminacy or lack of actual values). On the other, there are *stochastic fluctuations*, which are associated with, either, ensembles of similar systems (say, the grades of students in a class) or pertain to a single extended system and the spatiotemporal variation of a local quantity in it (e.g., the water level in a lake). This kind of confusion often occurs in connection with another prevalent source of misunderstandings, namely, the failure to distinguish between *proper* and *improper* mixtures.

Many of these problems can often be traced to the widespread propensity among physicists to ignore the elephant in the room: the *measurement problem* in quantum theory. In practice, the issue is reflected in the fact that the theory contains 2 rules determining the dynamics of the quantum state: the deterministic, time-reversible and unitary evolutionary rule, provided by the Schrödinger equation, and the stochastic, irreversible and non-unitary reduction rule associated with a measurement process. The problem, of course, is not so much that there are two rules, but that there in no unambiguous recipe specifying which one applies in each circumstance—i.e., precisely determining what kind of process constitutes a measurement. The issue has been faced by the community by adopting a variety of postures and strategies. Valuable guidance regarding the available options can be extracted from the result in Maudlin (1995), showing the following 3 premises to be, in conjunction, inconsistent:

1. The physical description given by the quantum state is complete.

2. Quantum evolution is always unitary.

3. Measurements always yield definite results.

The negation of 1 leads hidden-variable theories, that of 2 to spontaneous collapse models and that of 3 to Everettian interpretations. It is worth pointing out that, contrary to widespread belief, the measurement problem is not solved by decoherence, Okon and Sudarsky (2016a).

Next, we must discuss the way in which we approach the exploration of the quantum gravity regime. We refer to it as a *top-down* approach, in contrast with the more traditional *bottom-up* one, which starts by assuming one has a fundamental theory of quantum gravity at hand (say, string theory, loop quantum gravity or causal sets) and then works in attempts to apply such theory to deal with concrete problems, usually in regimes deemed of interest for representing concrete aspects of the "world out there". The regimes more often studied in these specific applications involve aspects of cosmology, black hole physics, etc. The top-down approach involves adopting an agnostic posture regarding the nature of the fundamental theory of gravitation, and thus spacetime, and focuses instead in pushing the application of our existing, well-tested theories to situations that seem to lie just beyond their standard domain of application. The idea is that, in so doing, one might be forced to introduce some relatively mild modifications which, if successful, could provide valuable clues about the nature of the more fundamental theory.

More specifically, the idea is to use GR and quantum field theory in curved spacetimes (i.e., semiclassical gravity) to address questions commonly expected to lie beyond their applicability. To do so, we will introduce minimal modifications to the general framework, as required by the specific problem and hand, all this while taking a definite position regarding how to deal with the measurement problem. Regarding the latter, we will focus on spontaneous collapse theories, on which there is a vast amount of work in the last 40 years (see Bassi and Ghirardi (2003); Bassi et al. (2013) for overviews). The basic idea is to construct a single dynamical equation to encompass, both, the unitary evolution and the collapse process.

The first viable theory of this sort is known as GRW, Ghirardi et al. (1986), and involves discrete spontaneous collapses of the wave function, separated by continuous periods in which the system is governed by the Schrödinger equation. Such a model introduces, on top of the Schrödinger equation, spontaneous reductions with rate λ, driving the state towards eigenstates of the position operator (smeared on scale r_c). The changes introduced are supposed to be very small when few particles are involved, but to become large when something like 10^{23} are entangled (and de-localized). For this, λ is chosen to be small enough, not to conflict with tests of quantum mechanics in the domain of subatomic physics, but big enough to result in rapid localization of “macroscopic objects". GRW suggested $\lambda \sim 10^{-16} sec^{-1}$ and $r_c \sim 10^{-5} cm$. These theories address the measurement problem successfully and are empirically viable (at least in non-relativistic regimes). They have been subject of experimental tests and some attractive specific versions have recently been ruled out.

Another interesting collapse model is Continuous Spontaneous Localization (CSL), Pearle (1989), which replaces the discontinuous jumps by a continuous stochastic evolution. The theory is defined by a modified Schrödinger equation, whose solution is

$$|\psi, t\rangle_w = \hat{\mathcal{T}} e^{-\int_0^t dt' \left[i\hat{H} + \frac{1}{4\lambda}[w(t') - 2\lambda \hat{A}]^2 \right]} |\psi, 0\rangle, \tag{1}$$

with $\hat{\mathcal{T}}$ is the time-ordering operator, $\hat{A}$ the so-called collapse operator and $w(t)$ a real, white-noise stochastic function whose probability is given by

$$PDw(t) \equiv {}_w\langle\psi, t|\psi, t\rangle_w \prod_{t_i=0}^{t} \frac{dw(t_i)}{\sqrt{2\pi\lambda/dt}}. \tag{2}$$

As in GRW, the collapse operator is taken to be related to a smeared position operator. Thus, in the context of many particle quantum mechanics (as a steep towards quantum field theory), one would use the smeared mass density operator, namely

$$\hat{M}(x) = \Sigma_i m_i \int d^3y \hat{\Psi}_i(y)^\dagger \hat{\Psi}_i(y) e^{-(||x-y||/r_c)^2}, \tag{3}$$

where the sum is over particle species. Moreover, as argued in Pearle and Squires (1994), there are good reasons to believe that λ, rather than a universal constant, should depend on the particle's mass.

In order to complete the theory, one must specify its ontology, i.e., to make explicit the connection between the formalism and what the theory says that exists “out there in the world". One of the most favored ontologies, adopted when working with non-relativistic spontaneous collapse theories, is the so-called mass density ontology, which takes the corresponding non-relativistic version of the expectation value of $\hat{M}(x)$ as providing the required connection between the formalism and the world.

Next, we expand on other aspects of our treatment. Spacetime will be provisionally described in classical terms—something which can be expected to be a good approximation in regimes where curvatures are small (i.e., $<< (1/l_{Planck})^2$) and quantum uncertainties are "not too large".[1] Matter, on the other hand is to be treated quantum mechanically (more specifically, when possible, we will use quantum field theory in curved spacetimes). These choices, quite naturally lead to the semiclassical Einstein equation, in which classical spacetime is sourced by the (renormalized) expectation value of the energy-momentum tensor for the relevant state of the matter fields. Since the Einstein tensor is identically divergence-free, the non-conservation of the expectation value of the energy-momentum tensor within collapse theories would seem to lead to inconsistencies.[2] However, the fact that we do not take this equations to be fundamental allows us to move forward.

As noted from the start, we regard the theoretical framework we are working with not as fundamental, as would be the case if one adopted a bottom-up approach, but as an approximate description with limited applicability. Thus, it is natural to take the view that, during collapses, the semiclassical Einstein equations are not valid. This would be analogous to studying, say, the Navier-Stokes equations for describing a fluid, but having in mind that such equations are not fundamental, but merely effective. In that case, one should not find it surprising that there are situations in which those equations fail to hold. Consider, for instance, a wave breaking on a shore, generating foam and other phenomena whose explanation would have to rely on the molecular dynamics underlying the very nature of the fluid. Clearly these effects are not something that one could expect the Navier-Stokes equations to account for. In an analogous way, we must regard semiclassical gravity as an effective description and expect that there will be situations in which the description will fail and where, presumably, a full theory of quantum gravity would be required to account for the phenomena. One such situation would correspond to what, at the effective level, we describe as a spontaneous collapse of the quantum state of the matter fields.

At the formal level, we rely on the notion of *Semiclassical Self-consistent Configuration* (SSC), defined as follows. The set $\{g_{\mu\nu}(x), \hat{\varphi}(x), \hat{\pi}(x), \mathcal{H}, |\xi\rangle \in \mathcal{H}\}$ represents a SSC iff $\hat{\varphi}(x)$, $\hat{\pi}(x)$ and $\mathcal{H}$ corresponds to a quantum field theory over a spacetime with metric $g_{\mu\nu}(x)$ and, moreover, the state $|\xi\rangle$ in $\mathcal{H}$ is such that $G_{\mu\nu}[g(x)] = 8\pi G\langle\xi|\hat{T}_{\mu\nu}[g(x), \hat{\varphi}(x), \hat{\pi}(x)]|\xi\rangle$.

Regarding collapse, we take them to be, not simple jumps from state to state within a SSC, but transitions from one SSC to another SSC. Thus, a spontaneous collapse on the quantum state of the matter field will be accompanied by a sudden change in the spacetime metric. Establishing exactly how this is supposed to occur implies providing matching conditions for spacetime and rules for determining—assuming a standard type of collapse theory,

[1] The question of when are such fluctuations too large is, in itself, a rather nontrivial question.

[2] Moreover, as mentioned above, in Maudlin et al. (2020) it is shown that all reasonable approaches to the measurement problem seem to lead to violations of $\nabla^a T_{ab} = 0$, for whatever object one tries to assign the role of $T_{ab}(x)$ within such an approach. That means that, for any of the available paths to deal with the measurement problem, this issue would arise.

involving jumps within a given Hilbert space— corresponding states in the new Hilbert space. It must be noted that, technically, all this is highly nontrivial (a preliminary proposal in this direction was outlined in Diez-Tejedor and Sudarsky (2012))—and things only get more complicated when considering continuous collapses, such as in CSL, a situation which, we can only hope, might be addressed by taking an appropriate continuous limit, staring from a version designed for theories involving discrete collapses.

In any case, in what follows we will ignore some of the complications required to do this precisely and work with effective methods that reflect the approach we have discussed. In the next couple of sections we show how this is can be done in some concrete applications, which are in themselves connected to nontrivial difficulties in our current physical understanding.

3 The seeds of cosmic structure

Contemporary cosmology includes inflation as one of its most attractive components. One of the biggest achievements of inflation is claimed to be the account for the emergence of the seeds of cosmic structure with the correct spectrum, as a result of "quantum fluctuations". However, at the conceptual level, the standard account is not truly satisfactory. Let us explain why.

The starting point of the analysis is a cosmological spacetime (in a particular gauge)

$$ds^2 = a^2(\eta)\{-(1+2\Psi)d\eta^2 + [(1-2\Psi)\delta_{ij} + h_{ij}]dx^i dx^j\}, \tag{4}$$

and matter represented by an inflaton field, written as $\phi = \phi_0(\eta) + \delta\phi$, with $\delta\phi, \Psi, h_{ij}$ small perturbations containing the spatial dependencies. The background (a, ϕ_0) is treated classically and assumed to be dominated by the inflaton potential slow roll regime, so ϕ_0 changes slowly and the scale factor is approximately given by $a(\eta) = -1/(\eta H_I)$. We set $a = 1$ at the present cosmological time and assume that the inflationary regime corresponds to η in $(-\mathcal{T}, \eta_0)$ with $\eta_0 < 0$. The perturbations are treated quantum mechanically and assumed to be characterized by the vacuum state (essentially the Bunch-Davies vacuum) $|0\rangle$. The idea is that inflation dilutes all preexisting features and drives all space dependent fields towards their vacuum states.

These quantum fields are also said to be characterized by their "quantum fluctuations". However, the sort of confusion discussed in the introduction arises as, in the standard treatment, those features are wrongly identified as the primordial inhomogeneities, which eventually evolved into all the structure in our universe: galaxies, stars planets, etc. However, note that, according to the inflationary picture, the universe was *strictly homogeneous and* *isotropic*, both in the sectors described classically and quantum mechanically, and the unitary dynamic cannot change that fact. How is it, then, that this fully homogeneous and isotropic scenario leads, at latter times, to the formation of galaxies, stars, planets, and life?

3.1 Dealing with the problem

In our picture, spacetime is treated classically, while the inflaton field is treated using quantum field theory in curved spacetime. Thus, quantum mechanically, the zero mode of the field, $\hat{\phi}_0$, is taken to be in a highly excited (and sharply peaked) state (see Diez-Tejedor and Sudarsky (2012)), while the space-dependent modes are in the vacuum state. The quantum state of the inflaton and the spacetime metric are then assumed to satisfy the semiclassical Einstein equations

$$G_{\mu\nu} = 8\pi G \langle \xi | \hat{T}_{\mu\nu} | \xi \rangle. \tag{5}$$

Under these conditions, one essentially obtains the standard behavior for the background, $a(\eta) = \frac{-1}{\eta H_I}$, and slow roll for $\langle \hat{\phi}_0 \rangle$ in $(-\mathcal{T}, \eta_0), \eta_0 < 0$.

We concentrate, next, on the $\vec{k} \neq 0$ modes. For their study, we rely on a effective procedure we have checked to give equivalent results as the SSC formalism. In conformity with standard assumptions, we take for the early stages of inflation $\eta = -\mathcal{T}$, the state to correspond to $|0\rangle$ and for the operators $\delta\hat{\phi}_k$, $\hat{\pi}_k$ to be characterized by Gaussian wave functions centered on 0, with uncertainties $\Delta\delta\phi_k$ and $\Delta\pi_k$. We also assume $\Psi = h_{ij} = 0$.

Next enters the collapse, which modifies the quantum state and the expectation values. We first assume that the collapse is described by a collapse theory adapted to the situation, in which collapses occur discretely and mode by mode. Our universe then corresponds to one specific realization of the stochastic evolution. We focus on the scalar metric perturbations, $\Psi(\eta, x)$, which characterize the CMB temperature fluctuations (and seeds of structure). The Fourier decomposition of the semiclassical Einstein Equations yield

$$-k^2 \Psi(\eta, \vec{k}) = \frac{4\pi G \phi_0'(\eta)}{a} \langle \hat{\pi}(\vec{k}, \eta) \rangle. \tag{6}$$

With reasonable choices for the details of the collapse theory, agreement with observations can be achieved, Cañate et al. (2013).

In a CSL version, one must select the collapse operator. Obvious choices are the field operator or the momentum conjugate operator, with $\lambda = \tilde{\lambda} k^{\pm 1}$ fixed by dimensional considerations, but $(-\nabla^2)^{-1/4} \hat{\pi}(\vec{x})$ or $(-\nabla^2)^{1/4} \hat{\phi}(\vec{x})$ can also be chosen. How to decide? We do not really know, but the fact that, in the non-relativistic limit, the mass density seems to offer a good possibility for the collapse operator, suggests to explore how these operators are related to operators constructed out of $\hat{T}_{ab}(x)$. Moreover, as we will discuss latter, there are good reasons to introduce curvature-dependent coefficients.

In any event, once that choice of collapse operator is made, the resulting prediction for the power spectrum for density inhomogeneities is

$$P_S(k) \sim (1/k^3)(1/\epsilon)(V/M_{Pl}^4)\tilde{\lambda}\mathcal{T} \quad (7)$$

Taking the GUT scale for the inflation potential and standard values for the slow-roll leads to agreement with observation for $\tilde{\lambda} \sim 10^{-5}MpC^{-1} \approx 10^{-19}sec^{-1}$. We note that the order of magnitude obtained is not very different from the one proposed in the context of GRW.

Treatments with a similar spirit can be found in Martin et al. (2012) and Das et al. (2013). We must mention, though, that the recent analysis in Martin and V. (2020) leads to the conclusion that the details of the CMB can be used to rule out some simple extrapolations of CSL theory, in which the collapse operator is taken as the matter density perturbation. In that case, a prediction density perturbation spectrum incompatible with observations is encountered. However, as discussed in Bengochea et al. (2020), it is not at all clear how to extrapolate the version of CSL that works in the low energy, many-particle regime (i.e., one based on a smeared mass density collapse operator) to the semiclassical realm under consideration.

4 The black hole Information puzzle

The discovery by S. Hawking that, as a result of quantum field theory effects, black holes emit thermal radiation, has had enormous repercussions in our understanding, not only of black hole physics, but also on broader aspects of the nature of our world. In particular, it is expected that, after settling into quasi-stationary configurations, black holes should start loosing mass and, eventually, essentially disappear, leaving behind only thermal radiation (although there is the possibility that they could leave a small remnant, which we will ignore in this discussion). All this, however, seems to lead to a problem since quantum theory requires unitary relations between initial and final states

One must note, though, that this demand only applies to states on Cauchy hypersurfaces. Still, people often modify the demand, requiring unitary relation between states in $\mathcal{I}^-$ and $\mathcal{I}^+$, leading to a puzzle. Such demand seems to us very hard to account for. The point is that a true paradox only arises when one assumes (among other things, as discussed in detail in Okon and Sudarsky (2018)), that quantum gravity cures the singularity (see Unruh and Wald (2017) or Maudlin (2017) for alternative positions on this issue). Only in that case, that lack of unitarity displayed by the Hawking evaporation of the black hole (assume no remnants) would seem to lead to a conflict with quantum mechanics. However, even that argument ignores something important. Even standard textbook quantum mechanics involves departures from unitarity in connection to measurements, and it is only under the assumption of purely unitary evolution that a conflict could arise.

In what follows we explore the issue from the perspective of spontaneous collapse theories, in which departures from unitary evolution are always present. We present a picture where the two kinds of departures from unitarity are unified, as first proposed in Okon and Sudarsky (2014). The proposal has been studied explicitly in the 2-D CGHS Model and schematically in 4-D Modak et al. (2015b,a) (for further discussion see Perez and Sudarsky (2022)).

4.1 The objective collapse point of view

As noted, we will consider a quantum-field-theory-in-curved-spacetime treatment for the matter fields. We take the field ξ as corresponding to the matter that will undergo gravitational collapse and help form the black hole, while ψ represents the quantum field initially taken to be in the vacuum state, and on which we will focus for consideration of the issue of information loss. For the unitary evolution, we will be using the Heisenberg picture, in which the state remains fixed, but the field operators depend on time (and space), $\hat{\psi}(x), \hat{\xi}(x)$. The effects of the spontaneous collapse theory will be treated as those of an interaction and for those we will use the interaction picture.

Consider first the characterization of the *in* region, before the black hole forms. There, the initial state schematically can be written as

$$|\Psi_{in}\rangle = |0_{in}\rangle_{\psi} \otimes |matt\rangle_{\xi}, \tag{8}$$

where $|matt\rangle_{\xi}$ represents the matter undergo gravitational collapse. One then describes the state of a quantum field ψ at late times, in terms of degrees of freedom *inside* and *outside* of the black hole. The initial vacuum state can then be written as

$$|0_{in}\rangle_{\psi} = \sum_{F_\alpha} C_{F_\alpha} |F_\alpha\rangle^{ext} \otimes |F_\alpha\rangle^{int} \tag{9}$$

where a particle state F_α consists of an arbitrary, but *finite*, number of particles (or individual mode excitations). Tracing over the interior degrees of freedom would lead to an improper thermal state, corresponding to the Hawking flux. The complete initial state can then be written schematically as

$$|\Psi_{in}\rangle = \sum_{F_\alpha} C_{F_\alpha} |F_\alpha\rangle^{ext} \otimes |F_\alpha\rangle^{int} \otimes |matt\rangle_\xi \tag{10}$$

The idea now is to consider the evolution of the initial state, employing a modified theory involving spontaneous collapse. For concreteness, we will consider a CSL-type theory. To do so, we introduce a foliation parametrized by τ, corresponding to $W^2 = \text{const.}$ in the inside (with W the Weyl tensor) and continue it "almost arbitrary" outside. This makes the collapse parameter an effective function of τ which, in fact, diverges as the region where the classical singularity would have been is approached.

The CSL equations can be generalized to drive collapse into a state of a joint eigenbasis of a set of commuting operators $\hat{A}^I$. For each $\hat{A}^I$ there will be one $w^I(t)$. In that case, we have

$$|\psi, t\rangle_w = \hat{\mathcal{T}} e^{-\int_0^t dt' \left[i\hat{H} + \frac{1}{4\lambda}\sum_I [w^I(t') - 2\lambda \hat{A}^I]^2\right]} |\psi, 0\rangle. \tag{11}$$

We call $\hat{A}^I$ the *set of collapse operators*. Then we make a simplifying choice: collapses lead to a state of definite number of particles in the inside region. Moreover, since we are working in the interaction picture, $\hat{H} \to 0$ in the above equation.

Next, w assume that the CSL collapse mechanism is amplified by the curvature of space-time. That is, that the rate of collapse λ depends on the Weyl tensor as follows

$$\lambda(W) = \lambda_0 \left[1 + \left(\frac{W^2}{\mu}\right)^\gamma\right], \tag{12}$$

where $W^2 = W_{abcd}W^{abcd}$, $\gamma > 1/2$ is a constant and μ provides an appropriate scale (R^2 in 2-D). Therefore, in the region of interest, we have $\lambda = \lambda(\tau)$. As a result, this evolution achieves, in the finite time to the singularity, what ordinary CSL achieves in infinite time, i.e. to drive the state to one of the eigenstates of the collapse operators.

Then, the effect of CSL on the initial state is to drive it to one of the eigenstates of the joint number operators. Thus, at a hypersurfaces Σ very close to the singularity, the state will be

$$|\Psi_{in,\tau}\rangle = N C_{F_\alpha} |F_\alpha\rangle^{ext} \otimes |F_\alpha\rangle^{int} \otimes |matt\rangle_\xi \tag{13}$$

Note that there is no summation and that the state is a pure. However, due to the stochastic nature of the evolution, we do not know which one.

Let us now consider the role of quantum gravity. As discussed above, we will assume that it resolves the singularity and leads, on the other side, to a reasonable spacetime. Moreover we will assume that it does not lead to large violations of the basic spacetime conservation laws. As a result of these assumptions, the effects of quantum gravity can be represented by the transformation

$$NC_{F_\alpha} \left|F_\alpha\right\rangle^{ext} \otimes \left|F_\alpha\right\rangle^{int} \otimes \left|matt\right\rangle_\xi \to NC_{F_\alpha} \left|F_\alpha\right\rangle^{ext} \otimes \left|0^{post-sing}\right\rangle,$$

where $\left|0^{post-singularity}\right\rangle$ represents a zero energy momentum state, corresponding to a trivial region of spacetime (we are ignoring possible small remnants). We end up, then, with a pure quantum state but, as we explained above, we do not know which one.

Let us consider, then, an ensemble of systems prepared in the same initial state, described by the pure density matrix

$$\rho(\tau_0) = \left|\Psi_{in}\right\rangle \left\langle\Psi_{in}\right| \tag{14}$$

Now, let us consider the CSL evolution of this density matrix, up to the hypersurface just before the singularity. Finally, let us make use of our assumptions about the effects of quantum gravity. The density matrix characterizing the ensemble after the would-be-singularity is

$$\rho^{Final} = N^2 \sum_F e^{-\frac{E_F}{T}} \left|F\right\rangle^{\rm out} \otimes \left|0^{post-sing}\right\rangle \left\langle F\right|^{\rm out} \otimes \left\langle 0^{post-sing}\right|, \tag{15}$$

$$= \left|0^{post-sing}\right\rangle \left\langle 0^{post-sing}\right| \otimes \rho^{out}_{Thermal} \tag{16}$$

We see, then, that, in the end, the ensemble is described by a proper thermal state on future null infinity, followed by an empty region. Therefore, information was in fact lost as a result of the general quantum evolution, but there is nothing paradoxical at all about that fact.

5 The Entropic arrow of Time

It has been argued extensively that, in order to account for the thermodynamic arrow of time in the context of time reversal invariant laws of physics, one needs to assume a very spacial initial state of the universe. In Penrose (1979), Penrose has proposed that such demand can take the form of a initial condition law restricting the initial state to be one with vanishing Weyl curvature (while allowing arbitrarily large R). He's arguments are motivated by the observation that, at late times, the entropy of the universe seems to be connected with very large black holes, while there are no indication of large white holes in the early universe—despite the very large curvatures associated with that regime. That kind of "initial condition" law seems quite different from all other laws we have encountered before (except, perhaps, for the constraint equations that appear in theories involving gauge invariance). The considerations described above, involving the measurement problem, the origin of structure and black hole information puzzle suggest a different possibility.

Let us look again at the CSL evolution law

$$|\psi, t\rangle_w = \hat{\mathcal{T}} e^{-\int_0^t dt' \left[i\hat{H} + \frac{1}{4\lambda}[w(t') - 2\lambda \hat{A}]^2 \right]} |\psi, 0\rangle. \tag{17}$$

Ordinarily, one considers that, in generic circumstances, the evolution of large systems would be dominated by the Hamiltonian component (with the collapse sector becoming most relevant in situations where only a few degrees of freedom play a crucial role). However, if λ grows with $W^2 = W^{abcd}W_{abcd}$, as we suggested above, then, in regimes where W^2 is large, the large scale evolution would be practically random. As a result, we have a picture in which we can have a universe evolving randomly for an indefinite "lapse of time", until some point where, just by chance, there is a jump into a state of an almost vanishing W^2. Thereafter, the evolution would be orderly and dominated by the Hamiltonian dynamics, with relatively small stochastic modifications (this idea was firsts put forward in Okon and Sudarsky (2016b)).

In such a universe, structure such as galaxies, stars, planets and life would only appear in the orderly regime, and creatures studying it would look back and find their past characterized by a regime with extremely small Weyl curvature. This would be a kind of dynamical realization of Penrose's proposal, framed within a unified scheme, capable of accounting for various other open issues in physics. We find this quite attractive. Surprisingly, on the other hand, the emerging picture cannot but remind us of some more traditional accounts of creation, like those in Genesis or the Greek Theogony.

6 Discussion

As we have seen, the approach we have been following seems to be quite promising in addressing various issues that arise in the gravity-quantum interface. However, it is worth pointing out that, at the same time, it has given rise to certain concerns. One of he most important ones relates to the issue of violation of the conservation of energy (and more specifically the local conservation of energy-momentum). The issue was already noted early in Banks et al. (1984), but further analysis in Unruh and Wald (1995) indicated the initial worries to be exaggerated. In fact, various versions of dynamical collapse theories have been constructed to ensure compatibility with experimental bounds. Still, as we noted, there is a potential problem of inconsistency, which we have been working on with the use of the SSC and gluing formalism. Recent progress along these lines has been obtained in Juárez-Aubry et al. (2018); Juárez-Aubry and Sudarsky (2020); Kay et al. (2023). On the other hand, the resulting formalism is rather complicated and dealing with more than a few collapse events often becomes completely impractical. It is important to point out, though, that recent work seems to suggest that "cumulative effects" of $\nabla^a \langle \hat{T_{ab}} \rangle \neq 0$ might account for the existence of dark energy (see Josset et al. (2017); Perez and Sudarsky (2019); Perez et al. (2021); Perez and Sudarsky (2021)).

Another key issue is the construction of a fully general-relativistic collapse theory. Some proposals in this direction have been put forward in Bedingham et al. (2016) and a similar analysis of the black hole evaporation and information loss has been carried out using a relativistic version of dynamical collapse theory developed by D. Bedingham, Bedingham (2011).

There are, of course, other open issues that require resolution. For instance, one should ensure that the unwanted foliation dependence is removed from the framework. In particular, without an appropriately defined prescription, the essential quantity $\langle \hat{T_{ab}}(x) \rangle$ could depend on the choice of hypersurface passing through x simply because, in the context of a spontaneous collapse theory, different quantum states would be associated with different hypersurfaces and infinitely many such hypersurfaces go trough the event x. It is natural to expect, though, that such problems would be eliminated by the construction of a fully relativistic collapse dynamics.

One more issue that must be dealt with in implementing the program we have outlined is the fact that the expectation value of the energy-momentum tensor is an object that requires renormalization. This is something well-understood in situations in which the spacetime geometry is given (often as a background) and requires, among other things, that the state in question is a Hadamard state. In our case, this needs to be viewed as a requirement on the collapse theory Juárez-Aubry et al. (2018), but becomes much more complex in situations in which one needs to solve at the same time for the spacetime, as would be the case in, say, constructing the SSC to which the system collapses. Relevant progress in dealing with this question has recently been achieved and reported in Kay et al. (2023). Another open questions relates to the nature of universal collapse operator. As noted, in the non-relativistic context, a natural choice seem to be the (smeared) mass density operator and it is not clear what would make a suitable general-relativistic substitute for that. One might guess that the role should be played by the energy-momentum tensor, but complications are likely to arise (besides those already mentioned) from the fact that the various components of that tensor do not commute.

On the other hand, it should be noted that the program has much more potential. There is the natural resolution of the lack of primordial gravity waves generated during inflation (which have been searched for in the form of B-modes, without any evidence for them so far, León et al. (2017, 2018)). The program also seems to offer a simple path to address the problem of eternal inflation, Lechuga Soliz and Sudarsky (2023), and to account for the anomalous low power at low l, León and Sudarsky (2015, 2012). Moreover, the introduction of collapses could diffuse the problem of time in canonical quantum gravity, as discussed in Okon and Sudarsky (2014). There is a large amount of work ahead to continue the exploration of this line of research. Of course, our approach could, in the future, be shown to be unviable. However, as noted by Sir Francis Bacon, when considering the scientific enterprise in general "truth emerges more readily from error than from confusion". We believe that ignoring the conceptual problems of quantum mechanics in the application of the theory to other domains can be a serious source of confusion, particularly when referring to situations beyond the laboratory, as the ones considered here.

References

Banks, T., Susskind, L., and Peskin, M. E. (1984). Difficulties for the evolution of pure states into mixed states. *Nucl. Phys.*, 44 B(234):125.

Bassi, A. and Ghirardi, G. (2003). Dynamical reduction models. *Physics Reports*, 379:257–426.

Bassi, A., Lochan, K., Satin, S., Singh, T., and Ulbricht, H. (2013). Models of wave-function collapse, underlying theories, and experimental tests. *Rev. Mod. Phys.*, 85:471.

Bedingham, D., Modak, S. K., and Sudarsky, D. (2016). Relativistic collapse dynamics and black hole information loss. *Phys. Rev. D*, 94(4):045009.

Bedingham, D. J. (2011). Relativistic state reduction dynamics. *Found.Phys.*, 41:41.

Bengochea, G., León, G., Pearle, P., and Sudarsky, D. (2020). Discussions about the landscape of possibilities for treatments of cosmic inflation involving continuous spontaneous localization models. *European Physical Journal C*, 80:1021.

Carlip, S. (2008). Is quantum gravity necessary? *Class. Quant. Grav.*, 25:154010.

Cañate, P., Pearle, P., and Sudarsky, D. (2013). Continuous spontaneous localization wave function collapse model as a mechanism for the emergence of cosmological asymmetries in inflation. *Phys. Rev. D*, 87:104024.

Das, S., Lochan, K., Sahu, S., and Singh, T. P. (2013). Quantum to classical transition of inflationary perturbations: Continuous spontaneous localization as a possible mechanism. *Phys. Rev. D*, 88:085020.

Diez-Tejedor, A. and Sudarsky, D. (2012). Towards a formal description of the collapse approach to the inflationary origin of the seeds of cosmic structure. *JCAP*, 045:1207.

Ghirardi, G., Rimini, A., and Weber, T. (1986). Unified dynamics for microscopic and macroscopic systems. *Physical Review D*, 34:470–491.

Huggett, N. and Callender, C. (2001). Why quantize gravity (or any other field for that matter)? *Phil. Sci.*, 68(3):S382–S394.

Josset, T., Perez, A., and Sudarsky, D. (2017). Dark energy as the weight of violating energy conservation. *Phys. Rev. Lett.*, 118:021102.

Juárez-Aubry, B. A., Kay, B. S., and Sudarsky, D. (2018). Generally covariant dynamical reduction models and the hadamard condition. *Phys. Rev. D*, 97:025010.

Juárez-Aubry, Miramontes, T. and Sudarsky, D. (2020). Semiclassical theories as initial value problems. *Journal of Mathematical Physics*, 61:032301.

Kay, B. S., Juárez-Aubry, B. A., Miramontes, T., and Sudarsky, D. (2023). Semiclassical theories as initial value problemsspontaneous collapse theories, and the initial value formulation of semiclassical gravity. *Journal of Cosmology and Astroparticle Physics*, 01:40.

Lechuga Soliz, R. L. and Sudarsky, D. (2023). On the issue of eternal inflation. *arXiv:2308.01383*.

León, G., Majhi, A., Okon, E., and Sudarsky, D. (2017). Reassessing the link between b-modes and inflation. *Phys. Rev. D*, 96:101301(R).

León, G., Majhi, A., Okon, E., and Sudarsky, D. (2018). Expectation of primordial gravity waves generated during inflation. *Phys. Rev. D*, 98(2):023512.

León, G. and Sudarsky, D. (2012). Novel possibility of nonstandard statistics in the inflationary spectrum of primordial inhomogeneities. *Sigma*, 8:024.

León, G. and Sudarsky, D. (2015). Origin of structure: Statistical characterization of the primordial density fluctuations and the collapse of the wave function. *Journal of Cosmology and Astroparticle Physics*, 06:020.

Martin, J. and V., V. (2020). Cosmic microwave background constraints cast a shadow on continuous spontaneous localization models. *Phys. Rev. Lett.*, 124:080402.

Martin, J., V., V., and Peter, P. (2012). Cosmological inflation and the quantum measurement problem. *Phys. Rev. D*, 86:103524.

Mattingly, J. (2005). Is quantum gravity necessary? In Kox, A. J. and Eisenstaedt, J., editors, *The Universe of General Relativity*, pages 325–338. Birkhäuser.

Mattingly, J. (2006). Why Epply and Hannah's thought experiment fails. *Phys. Rev. D*, 73:064025.

Maudlin, T. (1995). Three measurement problems. *Topoi*, 14.

Maudlin, T. (2017). (information) paradox lost. *arXiv:*, 1705.03541.

Maudlin, T., Okon, E., and Sudarsky, D. (2020). On the status of conservation laws in physics: Implications for semiclassical gravity. *Studies in History and Philosophy of Modern Physics*, 69:67–81.

Modak, S. K., Peña, I., and Sudarsky, D. (2015a). Loss of information in black hole evaporation with no paradox. *Gen. Rel. and Grav.*, 47:120.

Modak, S. K., Peña, I., and Sudarsky, D. (2015b). Non-paradoxical loss of information in of black hole evaporation in collapse theories. *Phys. Rev. D*, 91(12):124009.

Okon, E. and Sudarsky, D. (2014). Benefits of objective collapse models for cosmology and quantum gravity. *Found. Phys.*, 44:114–143.

Okon, E. and Sudarsky, D. (2016a). Less decoherence and more coherence in quantum gravity, inflationary cosmology and elsewhere. *Found. Phys.*, 46:852–879.

Okon, E. and Sudarsky, D. (2016b). A (not so?) novel explanation for the very special initial state of the universe. *Class. Quant. Grav.*

Okon, E. and Sudarsky, D. (2018). Losing stuff down a black hole. *Foundations of Physics*, 48:411–428.

Page, D. N. and Geilker, C. D. (1981). Indirect evidence for quantum gravity. *Phys. Rev. Lett.*, 47:979.

Pearle, P. (1989). Combining stochastic dynamical state-vector reduction with spontaneous localization. *Physical Review A*, 39:2277–2289.

Pearle, P. and Squires, E. (1994). Bound state excitation, nucleon decay experiments, and models of wave function collapse. *Phys.Rev. Lett.*, 73:1.

Penrose, R. (1979). Singularities and time-asymmetry. In Hawking, S. W. and Israel, W., editors, *General Relativity: An Einstein Centenary Survey*, pages 581–638. Cambridge University Press.

Perez, A. and Sudarsky, D. (2019). Dark energy from quantum gravity discreteness. *Phys. Rev. Lett.*, 122:221302.

Perez, A. and Sudarsky, D. (2021). Cosmological constraints on unimodular gravity models with diffusion. *Gen. Rel. and Grav.*, 53:40.

Perez, A. and Sudarsky, D. (2022). A dialog on the fate of information in black hole evaporation. In *Special Topic Collection Celebrating Sir Roger Penrose's Nobel Prize*. AVS Quantum Science, (AIP Press).

Perez, A., Sudarsky, D., and Wilson-Ewing, E. (2021). Resolving the h_0 tension with diffusion. *Gen. Rel. and Grav.*, 57:7.

Unruh, W. G. and Wald, R. M. (1995). On evolution laws taking pure states to mixed states in quantum field theory. *Phys.Rev. D*, 52:2176.

Unruh, W. G. and Wald, R. M. (2017). Information loss. *Rept. Prog. Phys.*, 80:092002.